A Deformable Model of Soap Film considering Physical Properties

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1 Introduction

Modeling of the soap film for a given boundary curve, called Plateau’s problem, is identical with constructing the surface of minimal area. Since mathematicians have dealt with this topic in 19th century, two methods have been widely used to solve the minimal surface problem. The first algorithm originated with solving the linear Dirichlet problem in [Pinkall and Polthier 1993], and another approach in [M. Desbrun 1999] was to evolve the surface via mean curvature flow. In order to describe the deformation of soap film based on dynamics, however, it is required to consider its physical properties as well as geometric properties. In this paper, we propose a physics-based model for the deformation of soap film through discrete differential geometry. It shows robust results for given boundaries as inputs.

2 The Proposed Model

Our proposed model is related to surface energy $E$ generated by deformation due to external force. The geometry of soap film at each time step is determined by the ordinary differential equation of motion $\mathbf{x} = -M^{-1} \nabla E(\mathbf{x})$ where $\mathbf{x}$ is the position vector of the vertices of the deformed shape, and $M$ is the mass matrix, $\nabla E(\mathbf{x})$ is the gradient of energy with respect to $\mathbf{x}$. It is essential to consider physical properties of soap film to define the surface energy. A soap film model is analogous to discrete thin shell model, but it has a ‘surface tension’. It makes the soap film into a minimal surface which has a boundary as its specified constraint. Surface tension $\sigma$ is the force per unit length, which is caused by the attraction between the liquid’s molecules by intermolecular forces. As the surface area increases after deformation, more molecules pull back to each other, and the surface becomes a minimal surface in equilibrium. We derive a surface energy based on surface tension in the following subsection.

Surface Energy  We define surface energy as a scalar function proportional to an area from the definition of surface tension $(N_{area} Hn^2)$. If we apply the concept of discrete differential geometry to discretize this physical energy, the surface energy of each vertex is the sum of energy within the Voronoi region around the vertex. Then the surface energy at vertex $\mathbf{x}$ is formulated as the multiplication of surface tension $\sigma$ and Voronoi area $A_v$. Particularly, if we observe the discrete mean curvature normal operator that is formulated as $\Delta_B = 2Hn = \lim_{A \to 0} \frac{\sum A_i}{A}$, the gradient descent of surface energy, i.e., negative gradient of area, is 3 DOF vector with the opposite direction of mean curvature normal $n$ and the magnitude proportional to the area $A$ and mean curvature $H$.

Additionally, the energy that resists the stretch plays an important role in simulating the motion, where it is dependent upon the elongation and material stiffness. Without this energy on an edge, we are faced with degeneracy such as self-intersection and overlap for a significant deformation. Finally, we formulate our surface energy as a summation over all vertices and edges as follows:

$$E = \sum_v \sigma A_v + \sum_e \frac{1}{2} k_e \| e \|^2$$  \hspace{1cm} (1)

where $v$ and $e$ are vertex and edge of the mesh respectively, $k_e$ is the stiffness of soap film material. Thus, the forces at vertex $\mathbf{x}$ is

$$F(\mathbf{x}) = -\nabla E(\mathbf{x}) = -2\sigma A_v Hn + \sum_{i \in N(\mathbf{x})} -k_e \| \mathbf{x} - \mathbf{x}_i \|$$  \hspace{1cm} (2)

where $N(\mathbf{x})$ is the set of 1-ring neighbor of $\mathbf{x}$.

3 Conclusion

We define the surface energy by physical properties of soap film and discrete differential geometry operator, and apply this energy to simulate its deformation. As shown in Figure 1, the proposed model describes the characteristics of soap film nicely. It is also possible to compute this in real-time and is easy to implement. Deformable models with the tiny vibration and combination, fracture of the soap bubble will be included in our future research.

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References